Math 10B with Professor Stankova
Worksheet, Discussion #13; Thursday, 3/1/2018
GSI name: Roy Zhao

1 Probability Distributions

1.1 Concepts

1.

Distribution	PMF	Example
Uniform	If $\#R(X) = n$, then $f(x) = \frac{1}{n}$ for all $x \in R(X)$.	Dice roll, $f(1) = f(2) = \cdots = f(6) = \frac{1}{6}$.
Bernoulli Trial	$f(0) = 1 - p$, $f(1) = p$	Flipping a biased coin
Binomial	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$	p is probability of success. Repeat n Bernoulli trials. Number of 6's rolled when rolling 10 die is $f(k) = \binom{10}{k}(1/6)^k(5/6)^{10-k}$.
Geometric	$f(k) = (1-p)^k p$	k failures and then a success.
Hyper-Geometric	$f(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	Counting the number of red balls I pick out of n balls drawn if there are m red balls out of N balls total.
Poisson	$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$	Count the number of babies born today if on average there are 3 babies born a day.

1.2 Examples

2. On average, there are 20 rainy days in Berkeley per year. What is the probability that this year, there are 30?

3. The probability of seeing a shiny Pokemon is approximately 1 in $10000 = 10^4$. What is the probability that I don't see any in my playthrough if I see 10^5 Pokemon total? (calculate both exactly and an approximation)

1.3 Problems

4. When a cell undergoes mitosis, the number of mutations that occurs is Poisson distributed and an average of 11 mutations occur. What is the probability that no more than 1 mutation occurs when a cell divides?

5. The number of chocolate chips in a cookie is Poisson distributed with an average of 15 chocolate chips. What is the probability that you pick up a cookie with only 10 chocolate chips in it?

6. The number of errors on a page is Poisson distributed with approximately 1 error per 100 pages of a book. What is the probability that a novel of 300 pages contains no errors?

7. Approximately 4 people are born every second. What is the probability that in a minute, there are 240 people born?

1.4 Extra Problems

8. When a cell undergoes mitosis, the number of mutations that occurs is Poisson distributed and an average of 8 mutations occur. What is the probability that no more than 1 mutation occurs when two cells divide?

9. The number of chocolate chips in a cookie is Poisson distributed with an average of 5 chocolate chips. What is the probability that you pick up a cookie with 10 chocolate chips in it?

10. The number of errors on a page is Poisson distributed with approximately 0.2 errors per 50 pages of a book. What is the probability that a novel of 300 pages contains no errors?

11. Approximately 4 people are born every second. What is the probability that in a minute, there are 80 people born?

2 Expected Value and Variance

2.1 Concepts

12.

Distribution	PMF	$E(X)$	Variance
Uniform	If $\#R(X) = n$, then $f(x) = \frac{1}{n}$ for all $x \in R(X)$.	$\sum_{i=1}^{n} \frac{x_i}{n}$	$\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{n}$
Bernoulli Trial	$f(0) = 1-p$, $f(1) = p$	p	$Var(X) = p(1-p)$
Binomial	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
Geometric	$f(k) = (1-p)^k p$	$\frac{1-p}{p}$	$Var(X) = \frac{1-p}{p^2}$
Hyper-Geometric	$f(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	$\frac{nm}{N}$	$\frac{nm(N-m)(N-n)}{N^2(N-1)}$
Poisson	$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$	λ	λ

The **Expected Value** is the weighted average of all the values the random variables can take on. By definition, it satisfies some properties:

- $E[c] = c$
- $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$ for **all** random variables
- $E[XY] = E[X]E[Y]$ for **independent** random variables.

The **Variance** is defined as $Var(X) = E((X - \mu)^2)$. An easier form is $E(X^2) - E(X)^2$. It satisfies some properties:

- $Var(c) = 0$
- $Var(cX) = c^2 Var(X)$
- $Var(X + Y) = Var(X) + V(Y)$ for **independent** random variables.

2.2 Examples

13. I flip a fair coin 5 times. What is the expected number of heads I flip and what is the variance?

14. I roll two fair 6 sided die. What is the expected value of their product?

15. I flip a fair coin 10 times. What is the expected number of pairs of consecutive heads I flip? (The sequence HHH has two pairs of consecutive heads)

2.3 Problems

16. True False The expected value of a random variable X is the value such that the PMF at that point is the largest.

17. True False The expected value of a random variable X always exists.

18. While pulling out of a box of cookies, what is the expected number of cookies I have to pull out before I pull out an oatmeal raisin if 20% of cookies are oatmeal raisin? What is the variance?

19. What is the expected number of aces I have when I draw 5 cards out of a deck?

20. In a safari, safari-keepers have caught and tagged 300 rhinos. On a safari, out of the 15 different rhinos you see, there are 5 of them expected to be tagged. How many rhinos are there at the safari?

21. In a class of 30 students, I split them up into 6 groups of 5 on Tuesday. Today, Thursday, I split them up again randomly. What is the expected number of people in your new group were in your old group on Tuesday?

22. In a class of 30 students, I split them up into 6 groups of 5. What is the expected number of days of splitting them up randomly into new groups of 5 before I split them up into the same groups again (assume that the groups are indistinguishable)?

2.4 Extra Problems

23. While pulling out of a box of cookies, what is the expected number of cookies I have to pull out before I pull out an oatmeal raisin if 15% of cookies are oatmeal raisin? What is the variance?

24. What is the expected number of kings I have when I draw 8 cards out of a deck?

25. In a safari, safari-keepers have caught and tagged 100 rhinos. On a safari, out of the 20 different rhinos you see, there are 8 of them expected to be tagged. How many rhinos are there at the safari?